Constructions, Definitions, Postulates, and Theorems

Constructions
- Altitude
- Angle Bisector
- Circumcenter
- Congruent Line Segments
- Congruent Triangles
- Orthocenter
- Parallel Line
- Perpendicular Bisector
- Right Angle
- Right Triangle
- Segment

Definitions
(Note: The following are the most commonly used definitions in proofs. For a complete list of definitions for the course, refer to the glossary.)

- Altitude: A perpendicular segment from a vertex of a triangle to its opposite side.
- Angle Bisector: A ray between two other rays that divides the angle into two equal measures..
- Bisector: A line or segment that divides another figure into two congruent parts.
- Circle: A set of all points in a plane that are a given distance from a point identified as the center.
- Complementary Angles: Two angles that add up to $90^\circ$.
- Congruent Angles: Two angles with exactly the same degree measure.
- Congruent Segments: Two segments with exactly the same length.
- Congruent Triangles: Two triangles are congruent if and only if their corresponding parts are congruent: Corresponding Parts of Congruent Triangles are Congruent (CPCTC).
• Diagonal: A line segment connecting non-adjacent vertices of a polygon.

• Diameter: A segment with endpoints on a circle that passes through the center of that same circle.

• Equilateral Triangle: A triangle where all three sides are equal in length.

• Height: The length of a perpendicular segment from a vertex of a polygon to its opposite side; also known as its altitude.

• Isosceles Triangle: A triangle where at least two sides are equal in length.

• Median: A segment from a vertex of a triangle to the midpoint of the opposite side.

• Midpoint: A point that divides a segment into two equal parts. The ordered pair of this point can be found by the midpoint formula: \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \).

• Obtuse Angle: An angle that measures more than 90°, but less than 180°.

• Parallel Lines: Two lines that lie within the same plane and never intersect. Parallel lines have the same slope.

• Perpendicular Lines: Two lines that intersect at 90° angles. Perpendicular lines have slopes that are the negative reciprocals of one another.

• Perpendicular Bisector: A line that divides a segment into two halves and intersects the segment at a 90° angle.

• Radius: Any segment from the center of a circle to any point on that circle.

• Right Angle: An angle that measures exactly 90°.

• Right Triangle: A triangle where one angle measures 90°.

• Semicircle: Half of a circle.

• Similar Triangles: Two triangles are similar if corresponding angles are congruent and corresponding sides are proportional.

• Supplementary Angles: Two angles that add up to 180°.

• Straight Angle: An angle that measures exactly 180°.

• Tangent: A line that intersects a circle in exactly one point. The tangent is perpendicular to the radius at the point of tangency.

Theorems and Postulates

Addition Property of Equality
For real numbers a, b, and c, if a = b, then a + c = b + c.

Additive Identity
The sum of any real number and zero is that same real number. In other words, for any real number a, a + 0 = a.
Alternate Exterior Angles Theorem
If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

Alternate Interior Angles Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Angle Addition Postulate
The measure of an angle created by two adjacent angles may be found by adding the measures of the two adjacent angles.

Angle-Angle (AA) Similarity Postulate
If two corresponding angles of two or more triangles are congruent, the triangles are similar.

Angle-Angle-Side (AAS) Postulate
If two angles and a non-included side are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.

Angle-Side-Angle (ASA) Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

Arc Addition Postulate
The measure of an arc created by two adjacent arcs may be found by adding the measures of the two adjacent arcs.

Area of a Square
The area of a square is a measurement representing the space within the interior of a square. It is found by the formula \( A = s^2 \) where \( A \) is the area and \( s \) is the length of a side.

Area of a Triangle
The area of a triangle is a measurement representing the space within the interior of a triangle. It is found by the formula \( A = \frac{1}{2} \cdot b \cdot h \) where \( A \) is the area, \( b \) is the length of the base, and \( h \) is the length of the height.

Associative Property of Addition
For real numbers \( a, b, \) and \( c \), \( a + (b + c) = (a + b) + c \).

Associative Property of Multiplication
For real numbers \( a, b, \) and \( c \), \( a \cdot (b \cdot c) = (a \cdot b) \cdot c \).

Commutative Property of Addition
For real numbers \( a \) and \( b \), \( a + b = b + a \).

Commutative Property of Multiplication
For real numbers \( a \) and \( b \), \( a \cdot b = b \cdot a \).
**Congruent Arcs and Chords Theorem**
Two minor arcs within the same circle or between congruent circles are congruent if and only if their corresponding chords are congruent.

**Congruent Inscribed Angles Theorem**
Two or more distinct inscribed angles that intercept the same arc, or congruent arcs, are congruent.

**Corresponding Angles Postulate**
If a transversal intersects two parallel lines, then corresponding angles are congruent.

**Cross Product Property**
\[ \frac{a}{b} = \frac{c}{d} \]
For real numbers \( a, b, c, \) and \( d, \) \( \frac{a}{b} = \frac{c}{d} \) is equivalent to \( a \cdot d = b \cdot c \) or \( ad = bc. \)

**Distance between Two Points Postulate**
The distance between two points can be found by taking the absolute value of the difference between the coordinates of the two points.

**Distributive Property**
For real numbers \( a, b, \) and \( c, \) \( a(b + c) = ab + ac. \)

**Division Property of Equality**
For real numbers, \( a, b, \) and \( c, \) if \( a = b \) and \( c \neq 0, \) then \( \frac{a}{b} = \frac{c}{d}. \)

**Exterior Angle to a Circle Theorem**
If two secants, two tangents, or a secant and a tangent intersect outside a circle, the measure of the created angle between them is one-half the absolute value of the difference of the measures of their intercepted arcs.

**Hypotenuse-Leg (HL) Theorem**
If two right triangles have congruent hypotenuses and corresponding, congruent leg lengths, the two right triangles are congruent.

**Identity Property of Division**
Any number divided by 1 results in the same number. For example, \( c \div 1 = c. \)

**Inscribed Angle to a Semicircle Theorem**
An inscribed angle that intercepts a semicircle is a right angle.

**Inscribed Angle Theorem**
The measure of an inscribed angle is equal to half the measure of its intercepted arc.

**Inscribed Quadrilateral Theorem**
The opposite angles of an inscribed quadrilateral to a circle are supplementary.
**Intersecting Chords Theorem**
The point of intersection between two chords in a circle creates two pairs of segments whose products are equal.

**Intersecting Lines Postulate**
If two lines intersect, then they intersect in exactly one point.

**Isosceles Triangle Theorem**
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

- **The Converse of the Isosceles Triangle Theorem**
  If two angles of a triangle are congruent, then the sides opposite those angles are congruent making the triangle an isosceles triangle.

**Midsegment of a Triangle Theorem**
A segment connecting the midpoints of two sides of a triangle is parallel to the third side and half its length.

**Multiplication Property of Equality**
For real numbers a, b, and c, if a = b, then ac = bc.

**Multiplicative Identity**
Multiplying any number by 1 produces that number. For example, b • 1 = b.

**Opposite Angle Theorem**
If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

**Parallel Postulate**
Given a line and a point not on that line, there exists only one line through the given point parallel to the given line.

**Perpendicular Diameters and Chords Theorem**
If a diameter is perpendicular to a chord, then the diameter bisects the chord and the minor arc between the endpoints of the chord.

**Pieces of Right Triangles Similarity Theorem**
If an altitude is drawn from the right angle of a right triangle, the two smaller triangles created are similar to one another and to the larger triangle.

- **First Corollary to the Pieces of Right Triangles Similarity Theorem**
  The length of the altitude from the right angle of a right triangle is the geometric mean between the segments of the hypotenuse created by the intersection of the altitude and the hypotenuse.

- **Second Corollary to the Pieces of Right Triangles Similarity Theorem**
  Each leg of a right triangle is the geometric mean between the hypotenuse and the segment of the hypotenuse created by the altitude adjacent to the given leg.
Properties of Kites

- Two pairs of adjacent, congruent sides.
- Nonvertex angles are congruent.
- Diagonals are perpendicular.
- The diagonal connecting the vertex angles bisects the diagonal connecting the nonvertex angles.
- The diagonal connecting the vertex angles bisects those angles.
- The angles created by the diagonal connecting the nonvertex angles are congruent above and below the diagonal.

Properties of Parallelograms

- Opposite sides are congruent and parallel.
- The diagonals bisect each other.
- Opposite angles are congruent.
- Consecutive angles are supplementary.

Properties of Rectangles

- Opposite sides are congruent and parallel.
- The diagonals bisect each other.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Each angle is a right angle.
- The diagonals are congruent.

Properties of a Rhombus

- Opposite sides are congruent and parallel.
- The diagonals bisect each other.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- All four sides are congruent.
- The diagonals are perpendicular.
- The diagonals are angle bisectors.

Properties of a Square
• Opposite sides are congruent and parallel.
• The diagonals bisect each other.
• Opposite angles are congruent.
• Consecutive angles are supplementary.
• Each angle is a right angle.
• The diagonals are congruent.
• All four sides are congruent.
• The diagonals are perpendicular.
• The diagonals are angle bisectors.

**Properties of a Trapezoid**

• One pair of parallel sides.
• Consecutive angles between the bases are supplementary.

**Pythagorean Theorem**
If a right triangle has sides a and b and hypotenuse c then \( a^2 + b^2 = c^2 \).

• **Converse of the Pythagorean Theorem**
  In a triangle with sides a, b, and c, if \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

**Reflexive Property of Equality**
For real number a, \( a = a \).

**Same-Side Interior Angles**
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

• **Converse of Same-Side Interior Angles Theorem**
  If the same-side interior angles formed by two lines and a transversal are supplementary, then the two lines are parallel.

**Secant-Tangent Intersection Theorem**
When a secant and tangent intersect at the point of tangency, the angles created at the point of intersection are half the measurement of the arcs they intersect.

**Segment Addition Postulate**
If point C is between points A and B, then \( AC + CB = AB \).

**Side-Angle-Side (SAS) Postulate**
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
**Side-Angle-Side Similarity Postulate**
If two or more triangles have corresponding, congruent angles and the sides that make up these angles are proportional, then the triangles are similar.

**Side-Side-Side (SSS) Postulate**
If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

**Side-Side-Side Similarity Theorem**
If two or more triangles have three corresponding, proportional sides, then the triangles are similar.

**Square Root Property of Equality**
For any real number $a$, $\sqrt{a^2} = a$.

**Substitution Property of Equality**
For real numbers $a$ and $b$, if $a = b$, then $a$ can replace $b$ in any expression and vice versa.

**Subtraction Property of Equality**
For real numbers $a$, $b$, and $c$, if $a = b$, then $a - c = b - c$.

**Supplementary Angles of a Trapezoid Theorem**
Consecutive angles between the two bases of a trapezoid are supplementary.

**Symmetric Property of Equality**
For real numbers $a$ and $b$, if $a = b$, then $b = a$.

**Transitive Property of Equality**
For real numbers $a$, $b$, and $c$, if $a = b$ and $b = c$, then $a = c$.

**Triangle Exterior Angle Theorem**
The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

**Triangle Inequality Theorem**
The sum of the lengths of any two sides of a triangle is greater than the third side.

- **Corollary to the Triangle Inequality Theorem**
The length of the third side of a triangle is less than the total and greater than the absolute value of the difference of the other two sides.

**Triangle Proportionality Theorem**
If a line is parallel to one side of a triangle and also intersects the other two sides, the line divides the sides proportionally.

**Triangle Sum Theorem**
The sum of the measures of the angles in a triangle is $180^\circ$. 

**Vertical Angles Theorem**
Vertical angles are congruent.